MOMENT CLOSURE METHODS FOR KINETIC EQUATIONS OF COMPLEX TRANSPORT PHENOMENA AND THEIR NUMERICAL SOLUTION (2020)

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Course Description

Moment closure methods are considered for **kinetic equations** governing a range of complex transport phenomena, including non-equilibrium, **rarefied, gaseous flow** phenomena as described by the Boltzmann kinetic equation, **multi-phase poly-disperse spray** behaviour described by the Williams-Boltzmann equation, and **radiative heat transfer** in non-gray, participating media as described by the radiative transfer equation. Moment closure methods offer a means of significant complexity reduction when constructing approximate solutions to such high-dimensional equations, providing a good compromise between computational efficiency and accuracy for many practical engineering applications. Closure techniques based on classical Grad-type methods, maximum-entropy considerations, as well as quadrature formulations will all be considered. The **numerical solution** of the resulting systems of hyperbolic moment equations that result from the moment closure strategy will also be examined.

Syllabus

1. **Introduction**
   - Microscopic Versus Macroscopic Descriptions
     - Kinetic Descriptions of Transport Phenomena
     - Statistical Nature of Theories
   - Brief History of Moment Closure Methods
     - Approximate Solution Methods
     - Complexity Reduction via Dimension Reduction
   - Exemplars
     - Boltzmann Equation
     - Williams-Boltzmann Equation
     - Radiative Transfer Equation (RTE)
   - Notation
2. **Kinetic Theory of Gases**
   - Distribution Function and Macroscopic Averages
     - Maxwell-Boltzmann Distribution

- Boltzmann Equation
  - Monatomic and Polyatomic Gases
  - Single and Multi-Component Gases
- Boltzmann Collision Integral
  - BGK (Relaxation-Time) Approximation
- Moments of the Distribution Function and Boltzmann Equation
- Maxwell's Equation of Change
  - Conservation and Non-Conservation Forms
- Boltzmann's H-Theorem
  - Equilibrium Distribution and Collisional Invariant Quantities

3. **Classical Method of Moments for Monatomic Gas**

- Grad's Method of Moments (Moment Closures)
  - 20-Moment Closure (Grad)
  - 13-Moment Closure (Grad)
  - 10-Moment Closure (Gaussian Closure)
  - 8-Moment Closure
  - 5-Moment Closure (Euler Equations)
- Moment Equations
  - Quasilinear Hyperbolic System of Equations with Relaxation
  - Moment Realizability
- Chapman-Enskog Method
  - Euler Equations
  - Navier-Stokes Equations
  - Burnett and Super-Burnett Equations
- Recovery of Navier-Stokes Equations
  - 13- and 20-Moment Models Contain Fluid-Limit Equations
- Order of Magnitude Approach

4. **Method of Moments for One-Dimensional Kinetic Theory**

- Representative One-Dimensional Kinetic Equation
- Chapman-Enskog Method and Fluid-Limit Solutions
  - "Euler" and "Navier-Stokes" Solutions
- Grad's Method of Moments
  - Approximate Form for Distribution Function
  - Moment Equations and Closing Flux
  - Moment Realizability and Hyperbolicity

- Maximum-Entropy Moment Closures
  - 1D Polykinetic Models
  - 2D Polydisperse, Polykinetic Models
- Applications
  - PTCs

7. **Method of Moments for Radiative Heat Transfer**
   - Radiative Transfer Equation (RTE)
     - Gray and Non-Gray Participating Media
   - Spherical Harmonic Approximations (P_N Closures)
     - P_1 and P_3 Moment Closures
   - Maximum-Entropy Moment Closures (M_N Closures)
     - M_1 and M_2 Moment Closures
   - Applications
     - Gray and Non-Gray Media
     - 1D and 2D problems
8. **Numerical Solution of Moment Equations**
   - Quasilinear Hyperbolic System of Equations with Relaxation
     - Godonov-Type Finite-Volume Methods
     - Discontinuous-Galerkin (DG) Finite-Element Methods
   - Second-Order Godunov Finite-Volume Method
     - Piecewise Limited Linear Reconstruction
     - Riemann Solver Based Flux Functions
   - Adaptive Mesh Refinement (AMR)
     - Anisotropic Block-Based AMR
9. **Computational Examples**: An additional session will be organized in which attendees are given example code and problems to explore the performance of various moment closures for 1D problems related to gaskinetic flows, disperse sprays, and radiative heat transfer.